

### **Case Study: Advertising Up-Front Market Sales Plan Generator**

TV1 Network, a television network, is receiving up-front market requests from clients who wish to purchase advertising slots for the next broadcast year. The up-front market is the sales period of two to three weeks that follows the announcement of the programming schedules for the year. TV1 Network sells about 80% of its air-time inventory during the up-front market.

Tables 1 and 2 show the prices and inventory for the airing of 30-second commercials on the shows scheduled for the season's first quarter. TV1 Network managers set the prices based on rating forecasts and demand estimates. Shows with high historical ratings are more desirable to advertisers, and they command higher prices. Also, certain weeks, like the weeks before the end-of-the-year holidays, have higher prices. As the up-front market progresses, managers periodically revise prices by considering new market knowledge. Similarly, the inventory gets adjusted as new deals with advertisers are closed.

Tomson & Tomson (T&T), a pharmaceutical company, has requested TV1 Network to formulate a complete schedule of commercials for its line of skin care products. The schedule of commercials translates into a sales plan for TV1 Network, and it must satisfy the following requirements specified by Tomson & Tomson:

1. Budget: The plan must make use of the full budget specified by the client. T&T decides on the budget so as to achieve a desired number of impressions through its advertising campaign. An impression is defined as the viewing of a commercial by an individual in the target population.
2. Show Mix: The commercials in the plan must be distributed over the various shows in the program schedule in accordance to the client's preference. T&T specifies its preference as the fraction of the total number of commercials to be aired with each show in the schedule. Table 3 shows T&T's show mix requirement.
3. Weekly Weighting: The commercials in the plan must be distributed over the weeks in the year in accordance to the client's preference. T&T specifies its preference as the fraction of the total number of its commercials to be aired during each week in the year. Table 4 shows T&T's weekly weighting requirement.

An additional consideration for TV1 Network when formulating an up-front market sales plan is the allocation of premium inventory. After the period for the up-front market is completed, high-paying customers will seek to buy remaining premium inventory in the period referred to as scatter market. TV1 management ranks the shows and weeks in the year by their importance for the scatter market, and attempts to satisfy the up-front market requests with the minimum use of premium inventory. Table 5 shows TV1 Network's inventory ranking.

#### **Model Formulation:**

The following model is part of an automatic up-front market sales plan generator system. During the up-front market, TV1 Network must generate hundreds of proposals like the one requested by Tomson & Tomson. The ability to generate proposals efficiently and correctly is key to TV1 Network's financial performance.

Let *SHOWS* be the set of shows on TV1 Network's season programming, and let *WEEKS* be the set of weeks in the planning horizon. A complete schedule of commercials is defined by integer variables  $x(s, w)$  specifying the number of commercials to be aired with each show in each week.

Because of the discrete nature of the problem, the show mix and weekly weighting requirements cannot be satisfied exactly, and they are better expressed as ranges. Let *SHOW\_WEIGHT\_LB(s)* and *SHOW\_WEIGHT\_UB(s)* define the show mix range requirement for show *s*, and let *WEEK\_WEIGHT\_LB(w)* and *WEEK\_WEIGHT\_UB(w)* define the weekly weighting range requirement for week *w*. For example, if the customer specified a show mix requirement of 10% for a given show *s*, the range will be  $SHOW\_WEIGHT\_LB(s) = (10 - \alpha)\%$

to  $SHOW\_WEIGHT\_UB(s) = (10+alpha)\%$ . The user-defined parameter  $alpha$  is such that it is acceptable for the customer if the requirement falls within the range.

Furthermore, in a situation of tight inventory, it may not be possible to satisfy the show mix and weekly weighting range constraints. We introduce slack variables and penalties associated with these variables to formulate the problem using goal programming. Let  $slack\_show\_lb(s)$ ,  $slack\_show\_ub(s)$ ,  $slack\_week\_lb(w)$ ,  $slack\_week\_ub(w)$  be the slack variables. The penalties  $PENALTY\_SHOW(s)$  and  $PENALTY\_WEEK(w)$  vary with the importance of the requirement. Let

$$units = \sum_{s \in SHOWS, w \in WEEKS} x(s, w) \quad (1)$$

be the total number of shows in the plan. We can now write the show mix and weekly weighting constraints as:

$$\forall s \in SHOWS : \sum_{w \in WEEKS} x(s, w) - SHOW\_MIX\_LB(s) * units + slack\_show\_lb(s) \geq 0 \quad (2)$$

$$\forall s \in SHOWS : \sum_{w \in WEEKS} x(s, w) - SHOW\_MIX\_UB(s) * units + slack\_show\_ub(s) \leq 0 \quad (3)$$

$$\forall w \in WEEKS :$$

$$\sum_{s \in SHOWS} x(s, w) - WEEK\_WEIGHT\_LB(s) * units + slack\_week\_lb(s) \geq 0 \quad (4)$$

$$\forall w \in WEEKS :$$

$$\sum_{s \in SHOWS} x(s, w) - WEEK\_WEIGHT\_UB(s) * units + slack\_week\_ub(s) \leq 0 \quad (5)$$

Where,

$$\forall s \in SHOWS : slack\_show\_lb(s) \geq 0 \quad (6)$$

$$\forall s \in SHOWS : slack\_show\_ub(s) \geq 0 \quad (7)$$

$$\forall w \in WEEKS : slack\_week\_lb(w) \geq 0 \quad (8)$$

$$\forall w \in WEEKS : slack\_week\_ub(w) \geq 0 \quad (9)$$

$$\forall s \in SHOWS, \forall w \in WEEKS : x(s, w) \in \mathbb{N} \quad (10)$$

Let  $PRICE(s, w)$ ,  $INVENTORY(s, w)$ , and  $BUDGET$  be the price and inventory per show and per week, and the budget specified by the client. The budget and inventory constraints follow:

$$budget = \sum_{s \in SHOWS, w \in WEEKS} PRICE(s, w) * x(s, w) \quad (11)$$

$$budget \geq BUDGET \quad (12)$$

$$\forall s \in SHOWS, \forall w \in WEEKS : x(s, w) \leq INVENTORY(s, w) \quad (13)$$

Let  $RANK(s, w)$  be the television network's inventory ranking value for the scatter market that is used to define the rank value for the plan:

$$rank\_value = \sum_{s \in SHOWS, w \in WEEKS} RANK(s, w) * x(s, w) \quad (14)$$

Finally, the objective function minimizes the weighted sum of the penalty associated with slack variables and the use of premium inventory, where *RANKWEIGHT* is used to establish the relative importance of the objectives.

$$penalty = \sum_{s \in SHOWS} PENALTY\_SHOW(s) * (slack\_show\_lb(s) + slack\_show\_ub(s)) + \sum_{w \in WEEKS} PENALTY\_WEEK(w) * (slack\_week\_lb(w) + slack\_week\_ub(w)) \quad (15)$$

$$minimize(penalty + RANKWEIGHT * rank\_value) \quad (16)$$

### Implementation

The following Mosel program implements the mathematical model given by lines (1) to (16) in the previous section. The Mosel code accepts parameters for the budget limits, for the penalties associated with the slack variables, for the relative weight of the objectives, and for forming the ranges for the customer requirements. Also, the parameter *DATAFILE* specifies the input data file, and the parameter *MAXTIME* limits the search time for the optimizer. Note that the input data is presented in Mosel-specific format, but this can be easily modified to obtain the data from a variety of database sources.

The upper bound on the budget is necessary for the case when the rank value objective is ignored. In that case, the only objective is to satisfy the show mix and weekly weighting requirements, and there may be feasible solutions that far exceed the budget.

Finally, the solution reporting code that is not included in this section can be examined by opening the file *tvnetwork.mos* in the Xpress-IVE.

```

model tvnetwork
  !up-front market sales plan generator model
  uses "moxprs"

  parameters
    BUDGET = 12900 !Budget lower bound
    BUDGET_UB = 15000 !Budget upper bound

    PENALTY_SHOW = 100 !Penalty for not meeting show mix requirement
    PENALTY_WEEK = 100 !Penalty for not meeting weekly weighting requirement

    RANKWEIGHT = 0.5 !Rank value weight in the objective function

    UB_FACTOR = .5 !Factors to define customer constraint ranges
    LB_FACTOR = .5

    DATAFILE = "tvnetwork.dat"
    MAXTIME = -10
  end-parameters

  declarations
    SHOWS: set of string
    WEEKS: set of string

    SHOW_MIX: array(SHOWS) of real
    WEEK_WEIGHT: array(WEEKS) of real

    SHOW_MIX_LB: array(SHOWS) of real
    SHOW_MIX_UB: array(SHOWS) of real

    WEEK_WEIGHT_LB: array(WEEKS) of real
    WEEK_WEIGHT_UB: array(WEEKS) of real

    PRICE: array(SHOWS, WEEKS) of real
    INVENTORY: array(SHOWS, WEEKS) of integer

    RANK: array(SHOWS, WEEKS) of real
  end-declarations

```

```

initializations from DATAFILE
SHOW_MIX WEEK_WEIGHT PRICE INVENTORY RANK
end-initializations

forall(s in SHOWS) do
SHOW_MIX_UB(s) := (SHOW_MIX(s) + UB_FACTOR)/100
SHOW_MIX_LB(s) := (SHOW_MIX(s) - LB_FACTOR)/100
end-do

forall(w in WEEKS) do
WEEK_WEIGHT_UB(w) := (WEEK_WEIGHT(w) + UB_FACTOR)/100
WEEK_WEIGHT_LB(w) := (WEEK_WEIGHT(w) - LB_FACTOR)/100
end-do

declarations
!decision variable
x: array(SHOWS, WEEKS) of mpvar

!slack variables
slack_show_lb: array(SHOWS) of mpvar
slack_show_ub: array(SHOWS) of mpvar
slack_week_lb: array(WEEKS) of mpvar
slack_week_ub: array(WEEKS) of mpvar

!dependant variables
budget: mpvar !total budget used in plan
units: mpvar !total commercial units in plan
rank_value: mpvar !total rank value in plan
end-declarations

forall(s in SHOWS, w in WEEKS) do
x(s,w) is_integer
end-do

!budget constraints
budget = sum(s in SHOWS, w in WEEKS) PRICE(s,w) * x(s,w)

budget >= BUDGET
budget <= BUDGET_UB

!Total number of commercials in plan
units = sum(s in SHOWS, w in WEEKS) x(s,w)

!Plan total rank value
rank_value = sum(s in SHOWS, w in WEEKS) RANK(s,w) * x(s,w)

!Show-mix constraints
forall(s in SHOWS) do
sum(w in WEEKS) x(s,w) - SHOW_MIX_LB(s) * units + slack_show_lb(s) >= 0
sum(w in WEEKS) x(s,w) - SHOW_MIX_UB(s) * units - slack_show_ub(s) <= 0
end-do

!Weekly weighting constraints
forall(w in WEEKS) do
sum(s in SHOWS) x(s,w) - WEEK_WEIGHT_LB(w) * units + slack_week_lb(w) >= 0
sum(s in SHOWS) x(s,w) - WEEK_WEIGHT_UB(w) * units - slack_week_ub(w) <= 0
end-do

!Inventory constraints
forall(s in SHOWS, w in WEEKS)
x(s,w) <= INVENTORY(s,w)

penalty := sum(s in SHOWS) PENALTY_SHOW * (slack_show_lb(s) + slack_show_ub(s)) +
sum(w in WEEKS) PENALTY_WEEK * (slack_week_lb(w) + slack_week_ub(w))

setparam("XPRS_MAXTIME", MAXTIME)
minimize(penalty + RANKWEIGHT * rank_value)

```

## Results

Figures 1 and 2 present two sales plan for T&T generated by the Mosel program with data provided. The plan in Table 1 was generated with the parameter RANKWEIGHT set to zero, thus ignoring the rank value objective. This plan uses a budget of \$14,940 that is close the budget upper bound, and a total of 40 commercial units. The total violation of customer requirements

adds up to 4 percent points, with one show mix range and 5 weekly weighting ranges that were not satisfied.

The plan in Figure 2 was generated with the parameter RANKWEIGHT set to 0.5. This plan sacrifices the satisfaction of the show mix and weekly weighting constraints in order to reduce the use of premium inventory. The budget used is \$12,900, equal to the budget specified by the customer. Note that the budget used is highly correlated with the use of premium inventory. The total number of commercial units is now 35. The total violation of customer requirements adds up to 7.9 percent points, with 9 show mix ranges and 8 weekly weighting ranges that were not satisfied.

The problems are solved by the Xpress-Optimizer, which finds feasible integer solutions in a few seconds. These are good quality solution as judged by the degree of satisfaction of the customer requirements and by the use of premium inventory. The differences between the two plans illustrate the trade-off between the television network objectives and the customer objectives. The ability to generate plans quickly facilitates the negotiation between the parties.

Budget used: 14940 (BUDGET: 12900/BUDGET\_UB: 15000)  
 Units used : 40  
 Range Constraints Deviation: 1.6 units/4%

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Soap A																
Home A																
Movie A					3							1				4
Sitcom A																
Kids A																
Sitcom B														1		1
Home B																
News 1								1								1
Kids B																
News 2					1			3							4	8
Cops A																
Teens A																
Teens B												1				1
Sitcom C																
Teens C																
Docs						1								1	1	3
News 3					1		2	1		1		3				8
Cops B							1									1
Soap B											2			2		4
Sitcom D																
Teens D						1							1			2
Soap C					1									2		3
Cops C																
Home C																
Sitcom E																
Sitcom F																
Movie B						1		1	1		1					4
Total Weekly	0	0	0	0	6	3	3	6	1	1	3	5	1	6	5	

Figure 1: Sales Plan for T&T with RANKWEIGHT = 0

Budget used: 12900 (BUDGET: 12900/BUDGET\_UB: 15000)  
 Units used : 35  
 Range Constraints Deviation: 2.75 units/7.85714%

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Soap A																
Home A																
Movie A						1	2									3
Sitcom A																
Kids A																
Sitcom B											1					1
Home B																
News 1														1		1
Kids B																
News 2					2					1	2			2		7
Cops A																
Teens A																
Teens B					1											1
Sitcom C																
Teens C																
Docs						1									1	2
News 3									1			4		1	1	7
Cops B														1		1
Soap B								2					1		1	4
Sitcom D																
Teens D					1										1	2
Soap C					1	1										2
Cops C																
Home C																
Sitcom E																
Sitcom F																
Movie B							1	3								4
Total Weekly	0	0	0	0	5	3	3	5	1	1	3	4	1	5	4	

Figure 2: Sales Plan for T&T with RANKWEIGHT = 0.5

### Conclusions

This case study presents a Mosel model for generating advertising up-front market sales plans. The model captures customer requirements about budget and about the distribution of commercials among shows and weeks, as well as television network use of premium inventory requirements. The formulation is a mixed integer goal program, which may be run with different parameters in order to generate plans that emphasize the various conflicting objectives.

Finally, the case study focuses on the essence of the model to generate up-front market sales plans, and it makes some simplifications with respect to production models used in practice. Some of the features of a more complex production model that can be implemented with Mosel follow:

Full year planning horizon: The TV1 Network model only considers the first quarter as the planning horizon. When considering the full year horizon, it is common to impose constraints about how the budget is to be distributed over the four quarters in the broadcast year.

Different commercial unit lengths: The TV1 Network model assumes that all commercials have a 30-second duration. Production models consider commercial unit lengths ranging from 15 seconds to 120 seconds. These models also allow for unit mix constraints specifying how the commercials in the plan should be distributed over the unit lengths of the commercials.

Alternative measuring units: The TV1 Network model formulates the show mix and weekly weighting requirements using data expressed as a fraction of the total number of commercials aired. Production models also accept these data as a percentage of the total number of impressions or as a percentage of total dollar amount. Budget constraints can also be expressed in terms of impressions.

	09/18	09/25	10/02	10/09	10/16	10/23	10/30	11/06	11/13	11/20	11/27	12/04	12/11	12/18	12/25
Soap A	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
Home A	4	4	4	4	4	4	3	3	4	3	4	4	4	4	4
Movie A	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
Sitcom A	4	4	4	4	5	5	4	4	4	4	3	4	4	4	4
Kids A	4	4	4	4	4	4	4	4	4	4	0	5	4	4	4
Sitcom B	4	4	4	4	4	4	4	4	5	4	4	3	4	4	4
Home B	4	4	4	4	4	4	4	1	1	4	4	4	4	4	4
News 1	4	4	4	4	4	5	2	4	3	4	0	4	4	4	4
Kids B	4	4	4	4	4	4	4	4	3	4	4	4	3	4	4
News 2	4	4	4	4	4	4	4	4	4	4	4	0	4	4	4
Cops A	4	4	4	4	4	4	3	4	2	4	3	4	4	4	4
Teens A	4	4	4	4	4	4	4	4	4	4	4	4	4	1	4
Teens B	4	4	4	4	4	4	2	5	2	4	3	4	4	4	3
Sitcom C	4	4	4	4	4	4	4	4	4	4	2	4	4	1	4
Teens C	4	4	4	4	4	5	3	0	1	3	4	5	5	4	4
Docs	4	4	4	4	4	4	3	4	4	1	1	4	4	4	1
News 3	4	4	4	4	4	4	5	1	4	2	4	4	4	4	4
Cops B	4	4	4	4	4	4	4	0	4	2	4	4	3	4	3
Soap B	4	4	4	4	4	3	4	4	5	4	4	4	4	4	4
Sitcom D	4	4	4	4	4	4	4	4	4	5	4	4	5	4	4
Teens D	4	4	4	4	4	4	4	4	4	3	1	1	4	4	1
Soap C	4	4	4	4	4	4	2	2	4	3	4	4	4	4	4
Cops C	4	4	4	4	4	4	5	4	0	2	4	3	4	4	4
Home C	4	4	4	4	4	2	4	4	4	0	4	1	4	4	4
Sitcom E	4	4	4	4	4	4	4	4	4	4	4	4	4	3	4
Sitcom F	4	4	4	4	4	4	2	4	4	4	4	4	4	4	4
Movie B	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 1: TV1 Network Inventory(units)

	09/18	09/25	10/02	10/09	10/16	10/23	10/30	11/06	11/13	11/20	11/27	12/04	12/11	12/18	12/25
Soap A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Movie A	360	360	360	360	360	360	360	420	420	420	480	480	480	480	360
Sitcom A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Kids A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
News 1	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Kids B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
News 2	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Teens A	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
Teens B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom C	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
Teens C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Docs	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
News 3	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Soap B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom D	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Teens D	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Soap C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom E	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom F	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Movie B	360	360	360	360	360	360	360	420	420	420	480	480	480	480	360

Table 2: TV1 Network Prices(\$)

SHOW	MIX
Soap A	
Home A	
Movie A	10
Sitcom A	
Kids A	
Sitcom B	2
Home B	
News 1	2
Kids B	
News 2	20
Cops A	
Teens A	
Teens B	3
Sitcom C	
Teens C	
Docs	7
News 3	20
Cops B	3
Soap B	10
Sitcom D	
Teens D	6
Soap C	7
Cops C	
Home C	
Sitcom E	
Sitcom F	
Movie B	10

Table 3: T&T Show Mix Requirement (percents)

WEEK	WEIGHTING
09/18	
09/25	
10/02	
10/09	
10/16	15
10/23	7.5
10/30	7.5
11/06	15
11/13	3.5
11/20	3.5
11/27	7.5
12/04	11
12/11	3.5
12/18	15
12/25	11

Table 4: T&T Weekly Weighting (percents)

	09/18	09/25	10/02	10/09	10/16	10/23	10/30	11/06	11/13	11/20	11/27	12/04	12/11	12/18	12/25
Soap A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Movie A	360	360	360	360	360	360	360	420	420	420	480	480	480	480	360
Sitcom A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Kids A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
News 1	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Kids B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
News 2	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops A	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Teens A	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
Teens B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom C	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
Teens C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Docs	450	450	450	450	450	450	450	525	525	525	600	600	600	600	450
News 3	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Soap B	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom D	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Teens D	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Soap C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Cops C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Home C	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom E	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Sitcom F	300	300	300	300	300	300	300	350	350	350	400	400	400	400	300
Movie B	360	360	360	360	360	360	360	420	420	420	480	480	480	480	360

Table 5: TV1 Network Inventory Ranking(\$)